# Myths and Truths about Risk-Neutral Scenarios

In this article we explore two myths about risk-neutral scenarios and replace the myths with truths. Note that these concepts apply only to risk-neutral and not real-world scenarios.

## Risk-Neutral Myth #1: The short rate will follow the forward rate

This idea comes from comparing two zero coupon bonds—one that matures at time *T* and another that matures at time *T*+1:

$$(1+y_T)^{-T} = P_T = E[CF_T * DF_T] = E[1 * DF_T]$$

$$(1+y_{T+1})^{-T+1} = P_{T+1} = E[CF_{T+1} * DF_{T+1}] = E[1 * DF_{T+1}]$$

where *y* = spot rate, *P* = Price, *CF* = cash flow and *DF* = discount factor.

Next, we take a ratio of these two equations:

$$(1+f_T)^{-1} = \frac{(1+y_T)^{-T}}{(1+y_{T+1})^{-T+1}} = \frac{E[DF_T]}{E[DF_{T+1}]}$$

where *f* = forward rate, *y* = spot rate and *DF* = discount factor.

Since the two discounts overlap through time *T*, we should be able to simplify this to:

$$(1+f_{T)})^{-1} = E[\int_T^{T+1} (1+y)^{-t}]$$

$$f_T = E[\int_T^{T+1} y]$$

where *f* = forward rate and *y* = spot rate curve.

This works in a deterministic environment; in fact, it’s the basis of the Certainty Equivalent scenario. However, it fails in a stochastic environment. To see why, let’s simplify the example. First, we switch to a discrete timestep model and use the 1-year yield to discount instead of integrating. If we focus only on the first timestep, we can simplify our formula, since the initial discount is fixed:

$$(1+f_1)^{-1} = E[(1+y_1)^{-1}]$$

where *f* = 1-year forward rate and *y* = 1-year spot rate at end of first simulation period.

However, since we are doing stochastic simulations, we need to apply Jensen’s inequality.

$E[1 / X] >= 1 / E[X]$

if $X > 0$

The right and left-hand sides of the above inequality will be equal only when *X* is deterministic. If *X* is a stochastic variable, we need to respect Jensen's inequality which results in the following relationships:

$$E[(1+y_1)^{-1}] \geq \frac{1}{E[(1+y_1)]}$$

$$(1+f_1)^{-1} \geq \frac{1}{E[(1+y_1)]}$$

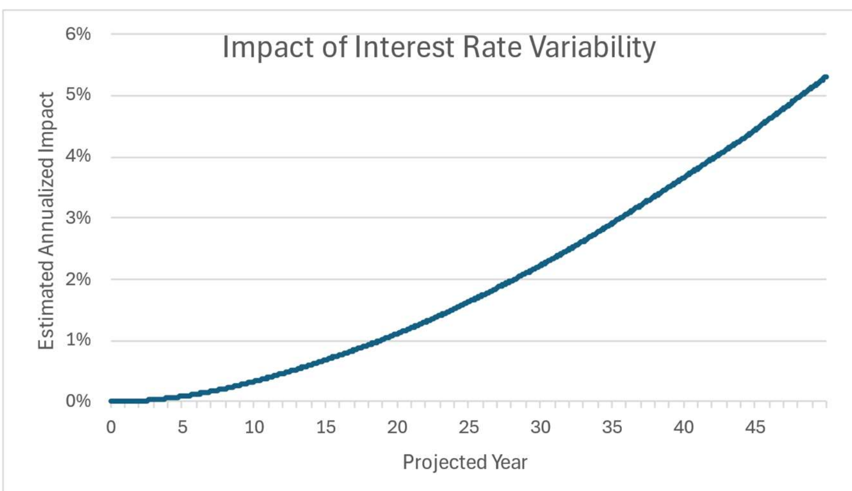
$$E[(1+y_1)] \geq (1+f_1)$$

where *y* = 1-year spot rate at end of first simulation period and *f* = 1-year forward rate.

**Jensen's Inequality:** Simply put, the average of a ratio is not equal to the ratio of the average. In our application, the average of 1/Discount is greater than 1/average Discount. Mathematically, E[1/*x*]>1/E[*x*], where *x* is the stochastic risk-neutral discount factor. Practically, the inequality is due to the variability in stochastic interest rates.

The right and left-hand sides of the above inequality will be equal only if the scenarios are fixed (i.e. there is no variance). Figure 1 shows the impact of the variability introduced in stochastic scenarios.

Figure 1: Difference for 1Q 2025 Scenarios




*Prepared by Conning, Inc. ©2025 Conning, Inc. Source: GEMS® Economic Scenario Generator.*

The discount factor used in this analysis is the risk-neutral discount factor in GEMS®, which is based on an overnight rate.

With that insight, we can correct the myth and state the following truth:

**Risk-Neutral Truth #1: The short rate will follow the forward rate *plus an adjustment for interest rate volatility***

## Risk-Neutral Myth #2: All asset classes will have the same expected return

This idea comes from a corollary of the pricing formula:

$$P = E[MV_T * DF_T]$$

where *P* = price, *MV* = market value, *DF* = discount factor and cash flows get reinvested in the asset.

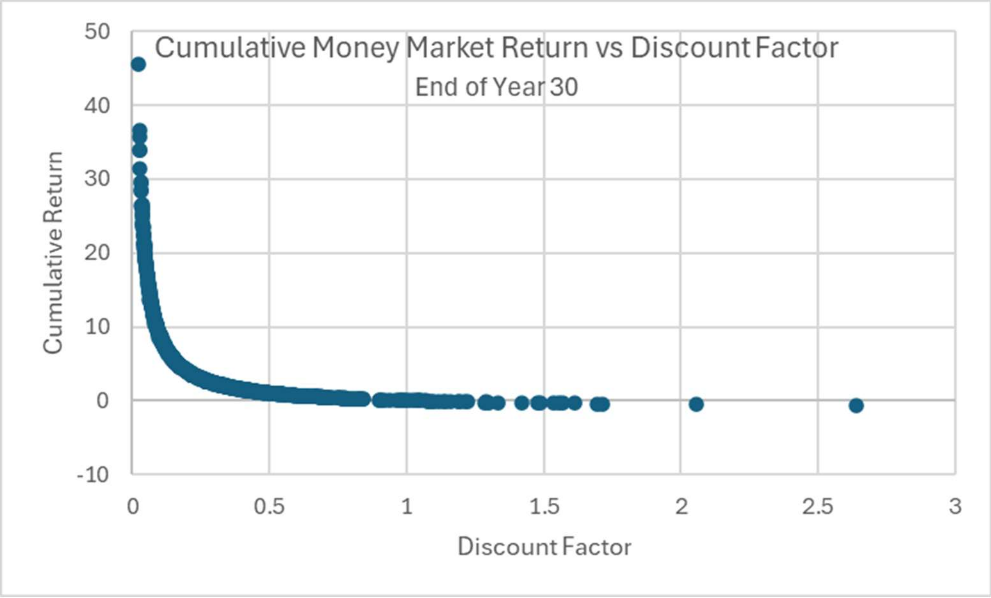
This can be extended into the relationship below, which is frequently referred to as the Martingale test:

$$1 = E\left[\left(\frac{MV_T}{P_0}\right) * DF_T\right] = E[(1 + CR_T) * DF_T]$$

where *P* = price, *MV* = market value, *DF* = discount factor and *CR* = cumulative total return.

It is easy to assume that this implies that the expected cumulative returns should be the same across all asset classes. Specifically, we might assume that returns would match appropriate spot rates since those cumulative returns are fixed. This would be true if returns and spot rates were independent. However, that is not the case for fixed income asset classes, since both the return and discount depend on yields. Figure 2 shows a clear relationship between cumulative money market returns and the discount factor.

Figure 2: Cumulative Money Market Return vs. Discount Factor




*Based on 1,000 scenarios as of 3/31/2025. Prepared by Conning, Inc.*


Since the discount factor is an accumulation of an overnight investment, the expected return on all asset classes will be aligned with the short rate. If we apply the Martingale concept to a *T*-year spot rate over a *T*-year projection period, we see that (1 + Cumulative Return) is fixed and equal to $e^{y_T * T}$. That means that $E[DF_T] = e^{y_T * T}$, which is how it is linked to the initial yield curve.

Figure 3: Discount Factor Linkage to Initial Price Curve

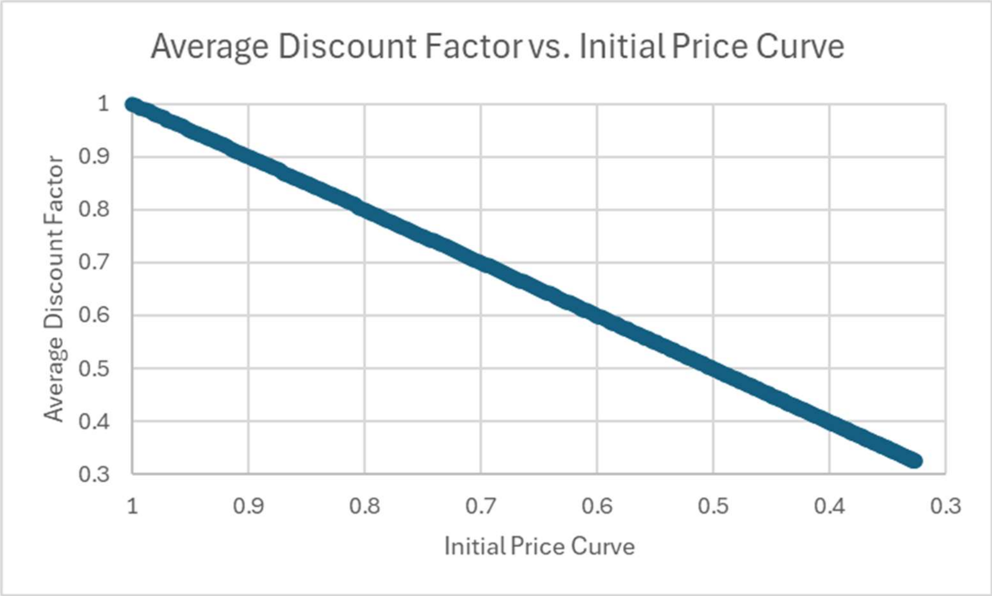



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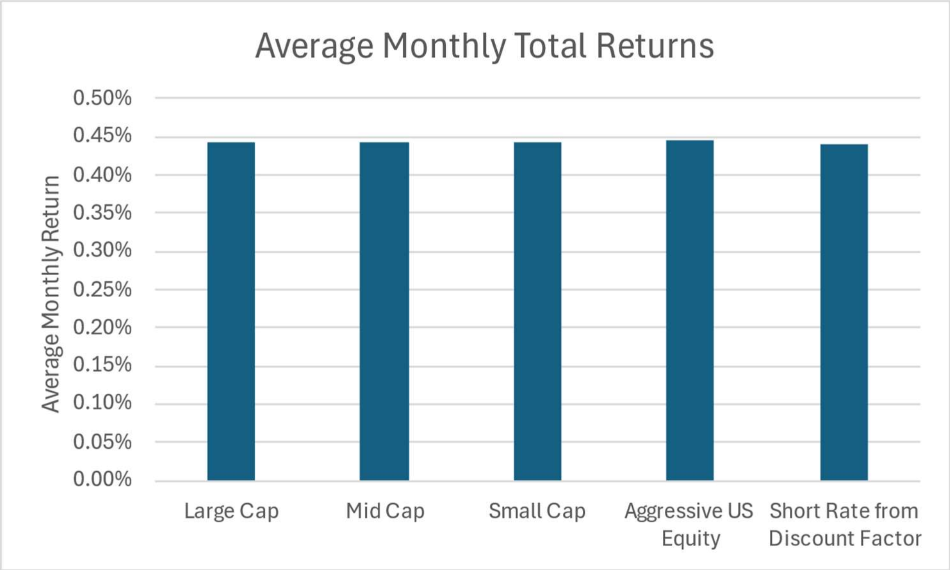
Figure 3 shows that the average discount factors are consistent with the initial price curve. The average discount factors are shown for a 30-year time horizon and the initial price curve is shown out to a 30-year tenor.

For the native equities in GEMS® (e.g. Large Cap, Mid Cap, etc.), the expected total return is short rate + equity-related variability. Since the equity related variability is independent of Treasury yields and nearly independent from one period to the next, we should expect that *E*[*Equity Total Return*] ~ *E*[*Short Rate*] for all the indices. It turns out that the equity volatility, which changes with the market environment, does not have much impact on these expected returns.

Figure 4: Average of Monthly Total Returns over 50-year Projection

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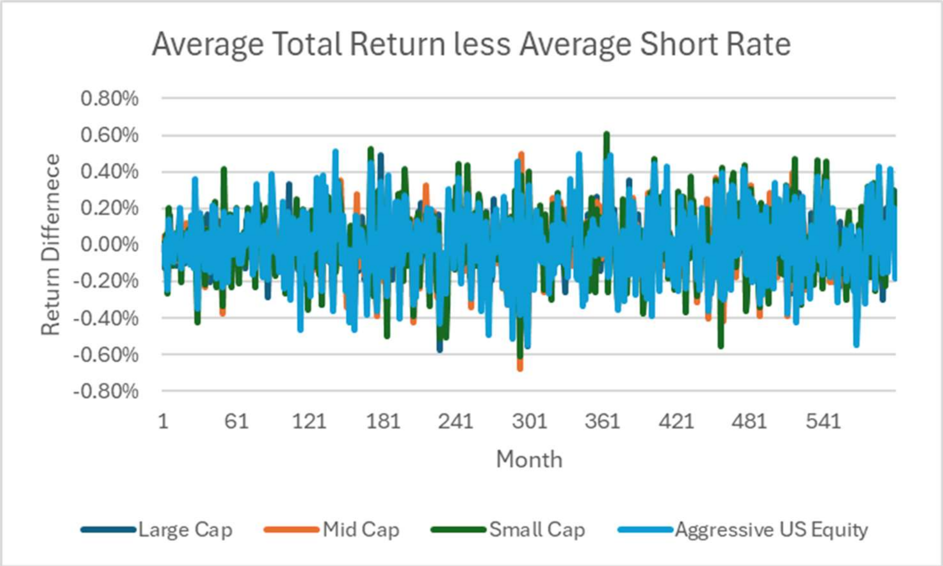
While the arithmetic average returns are equal, there can still be month-to-month variations in the average returns. Figure 5 shows the monthly average returns in excess of the short rate. While the average difference is zero, there is variability consistent with the underlying index volatility.

Figure 5: Average Total Return Less Average Short Rate

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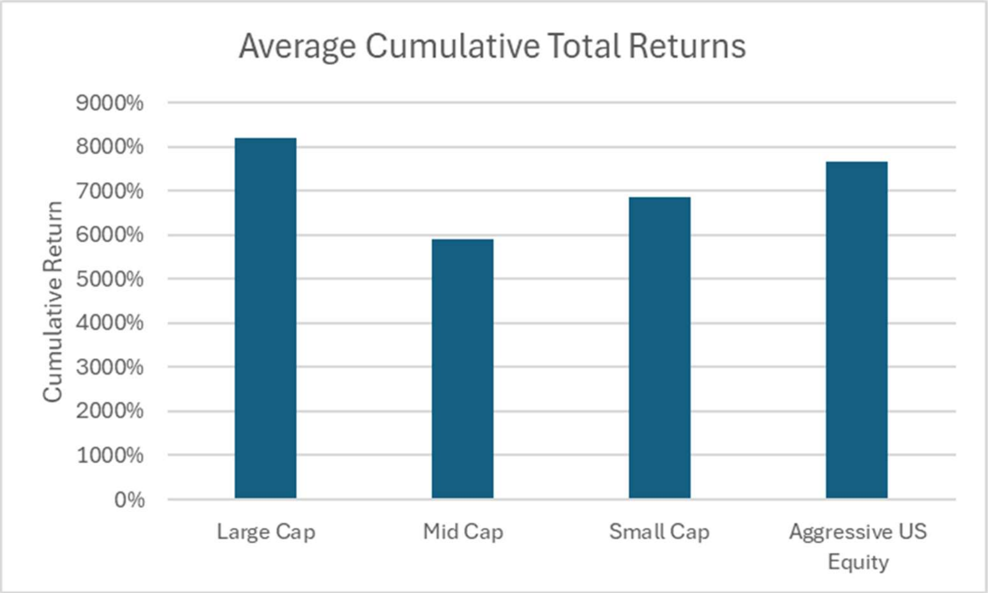
Because of this volatility, the expected cumulative returns will not be equal.

Figure 6: Average Cumulative Returns after 50 Years

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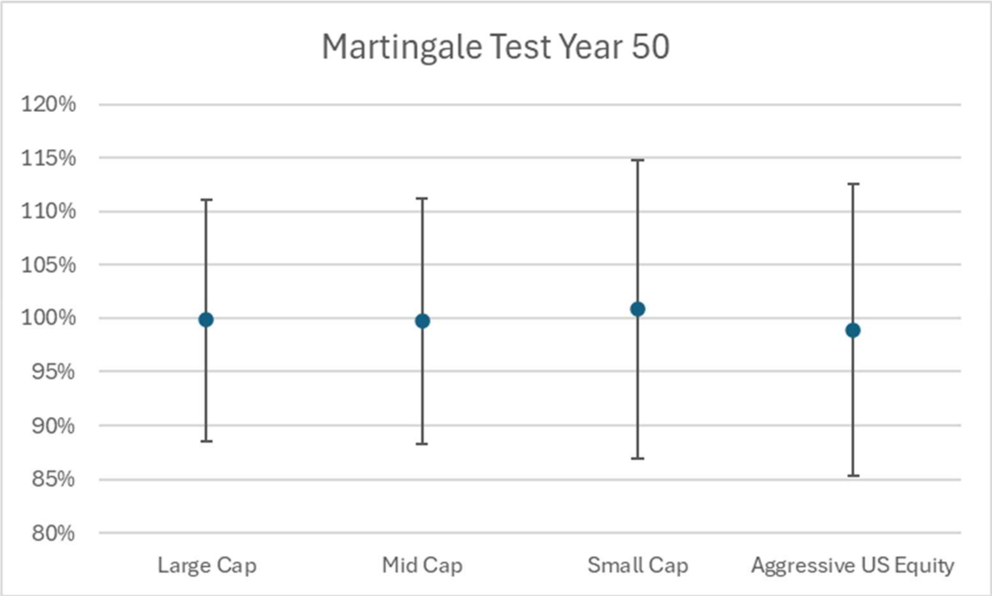
However, the expected discounted values of (1+*Cumulative Return*) will be the same and equal to 1.0, which brings us back to the Martingale test. As shown in Figure 6, the average discounted value of each of the equity indices is 1.0. While the results are shown only at the end of year 50, the same result is observed at every intermediate horizon. Because we need to recognize the sampling error in our 1,000-scenario example, the upper and lower error bars show the 95% confidence interval for the test statistic.

Figure 6: Martingale Test at the end of Year 50

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Recall that the Martingale test is based on total returns. For equity asset classes, dividend yields and prices are part of the equity calibration. Therefore, price returns can differ across different equity asset classes.

Armed with this additional information, we can clarify the myth and state the following truth:

Risk-Neutral Truth #2: All asset classes will have the same expected *arithmetic monthly total* return. *Price and income returns can be different. While cumulative returns can also be different, average discounted values will be the same* and *should be around 1.0.*

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